

1 Introduction

How much poverty is there?

Is poverty increasing or decreasing?

To answer such seemingly simple questions, analysts must choose (1) a criterion for deciding whether an individual is poor (such as whether his or her income is below the poverty line) and (2) an index which summarizes the amount of poverty in a society. This paper concerns the second issue, building on the contribution of Sen (1976).

Some indices of poverty (such as the poverty rate) are easy to understand but often misleading. Some others have desirable ethical properties but are rarely used in policy debates because of their complexity. When proposing an index of poverty, research

in terms of social welfare ev

common multiplicative decomposability of the Sen indices suggests that the decompositions must have similar useful and intuitive geometric interpretations.

In this paper, we examine the common underlying social evaluation function, multiplicative decomposability, and geometric interpretations of the Sen indices. We show that: (1) the Sen indices are based on a set of well justified and commonly agreed axioms; (2) the Sen indices share a common Gini social evaluation function, in which the social welfare is evaluated as the rank-weighted "average" of

number of the poor be .

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transfer of income is poor.¹³

(6) Contin

out of poverty will not change the average poverty gap ratio.

These

with $\forall_G(\mathbf{y}) = \forall_G$

SEF.²⁰ In the de $\bar{}$ nition, the EDEI function

[see Chakravarty (1997)]. Equation (15) provides a mathematical structure based on which one can see why the SST index

Proposition 2 The SST index has the following multiplicative decomposition:

$$SST(\mathbf{y}) = \bar{x}_p (1 + \mathbf{x}), \quad (18)$$

where \mathbf{x} has elements in non-decreasing order.

Proof: Using the relationship between \bar{x}_p and \mathbf{x}

where the poverty gap ratio vector of the poor, \mathbf{x}_p , has elements in non-increasing order. It is known that $\mathbf{x} = \mathbf{x}_p$. From equation (21) we get

$$(\mathbf{x}) = 1 - \frac{1}{n} \sum_{i=1}^q (2 - 2 + 1) \mathbf{x}_p \quad (23)$$

It can be further rewritten as

$$(\mathbf{x}) = \frac{1}{n} \left(1 - \frac{1}{2} \sum_{i=1}^q \mathbf{x}_p \right)$$

Proposition 4 The S index is bounded above by the SST index, i.e.,

$s(\dots)$

3.3 Similar Geometric Interpretations

The S index permits a simpler geometric interpretation that is somewhat different from that of Sen (1976) but is quite close to that of the SST index proposed by Shorrocks (1995). For comparison purpose, we also present and interpret the SST index geometrically

Note again that the relative deprivation measure is $r_i =$

line linking the origin 0 and the point (x_p, y_p) is area A_0 . Thus,

$$A_0 = \frac{1}{2} x_p y_p \quad (34)$$

$$A_c = \frac{1}{2} x_p^2 \quad (35)$$

Area A_0 can be computed from the fact that the Gini index of poverty gap ratios of the poor is given by²³

$$G(x_p) = \frac{A_0}{A_c} = \frac{A_0}{A_c} \quad (36)$$

Using equations (35) and (36) yields

$$A_0 = A_c G(x_p) = \frac{1}{2} x_p^2 G(x_p)$$

The S index is simply the ratio of the sum of areas A_0 and A_c to area A_0 , i.e.,

$$S(y_p)$$

and

$$\text{Area } B = (1 -) \pi$$

among all working-age households, due to the rising prop

both the working and workless households, a decrease in the poverty rate for the workless households, and an increase in the poverty rate for the working households; and (2) an increase in the percentage of workless households.

The 1974 data in Figure 5 also illustrate the importance of the relative si⁻e4r

and one plus the Gini index of the poverty gap ratios as follows:

$$\text{The S index} = \text{poverty rate}$$

Economics Working Paper 97{03, Dalhousie University, Halifax, Nova Scotia, Canada.

- [24] Osberg, L. and K. Xu (1999) 'Poverty intensity | How well do Canadian provinces compare?' *Canadian Public Policy*, 25(2), 1{17**
- [25] Osberg, L. and K. Xu (2000) 'International comparison of poverty intensity: Index decomposition and bootstrap inference,' *Journal of Human Resources*, 35, 51{81**

[36]

Table 1: Poverty Trend: Households with Head Less Than 65 years of Age in the United Kingdom

Year	Hhds %	Pov. Intensity %	Pov. Rate %	Pov. Gap %	1 + G
Panel A: All Working-Age Households					
1974	100.0	2.3	5.8	19.9	1.969
1979	100.0	1.5	3.1	23.6	1.982
1986	100.0	5.2	8.7	30.4	1.958
1991	100.0	6.4	12.8	25.7	1.929
1995	100.0	7.2	13.3	28.1	1.934
Panel B: Working Households					
1974	95.3	1.0			

Figure 1: Deprivation Profile

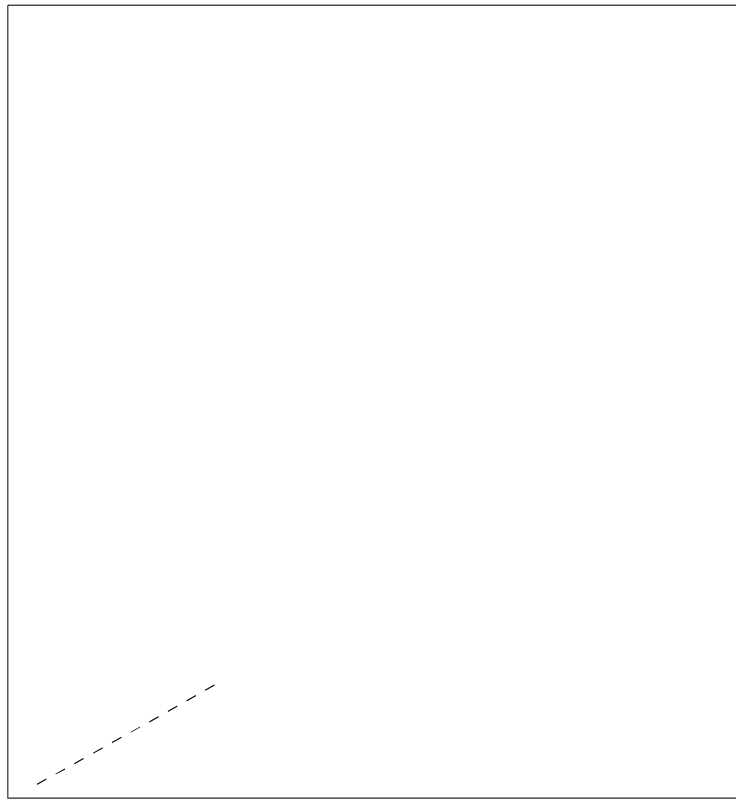


Figure 2: Geometric Interpretation of the S Index

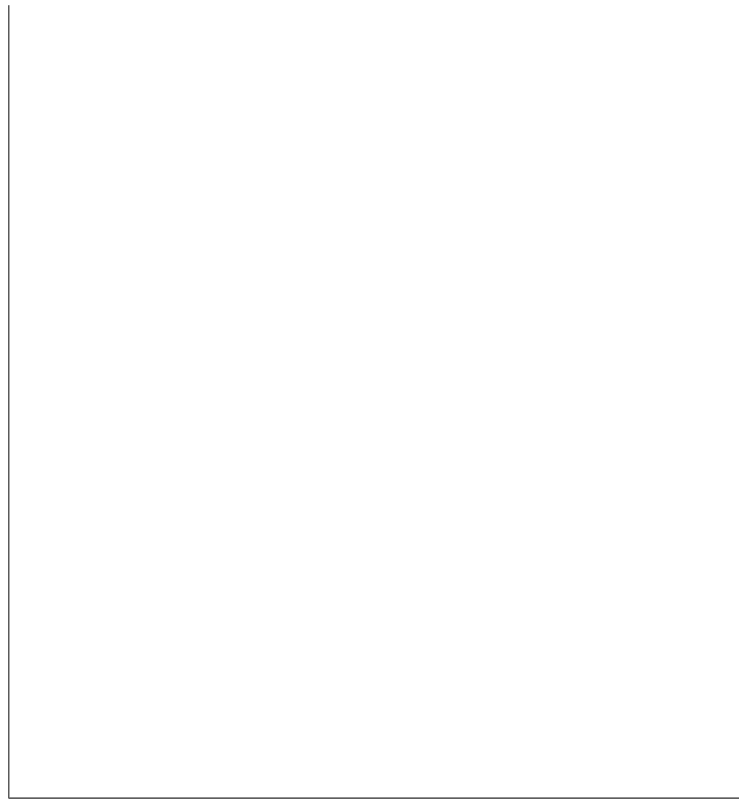


Figure 3: Geometric Interpretation of the SST Index

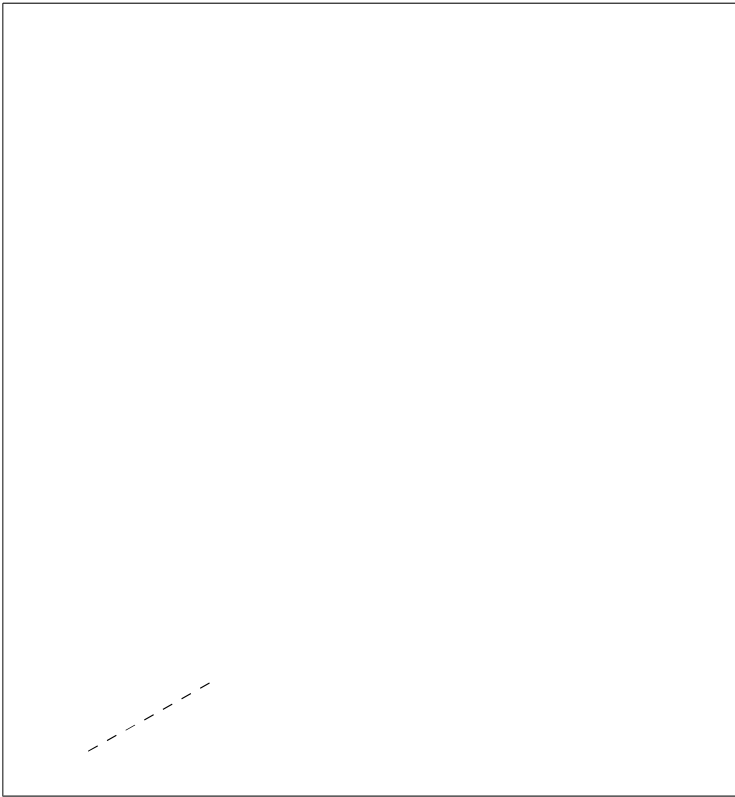


Figure 4:

Figure 5: Po (P)514.20 Td (P205 (P)514v20 Td 8448o (P)514ert879 03.962o (P)514y87

